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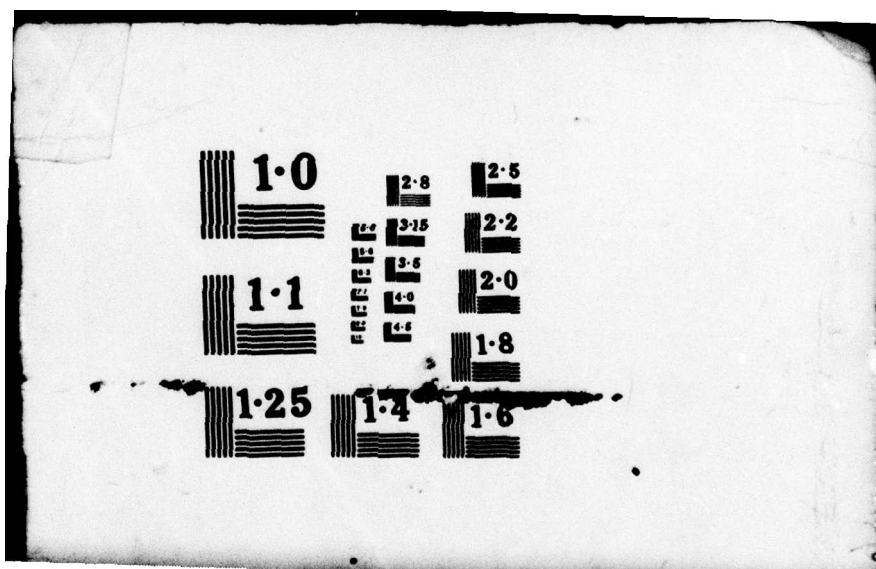UNCLASSIFIED   AFOSR-TR-79-0938   NL
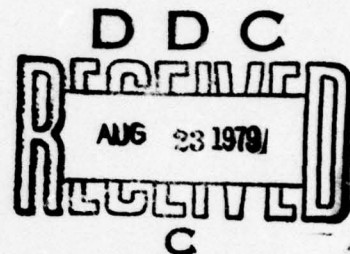
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Annual Technical Report
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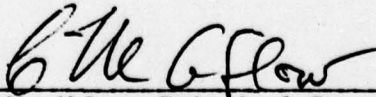TWO-DIMENSIONAL MESH ADJUSTMENT ALGORITHM

Prepared for:

Director, Mathematical and Informational Science
AIR FORCE OFFICE OF SCIENTIFIC RESEARCH/NM
Building 410
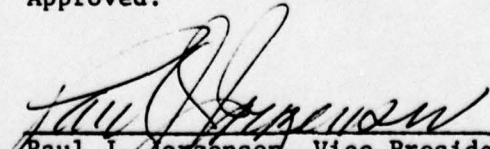Bolling Air Force Base, D.C.   20332

Attn: Captain Carl Edward Oliver
        Program Manager

Prepared by:

C. M. Ablow, Staff Scientist
S. Schechter, Senior Research Mathematician
Physical Sciences Division

C. M. Ablow, Principal Investigator

Approved:

Paul J. Jorgensen, Vice President
Physical and Life Sciences Group

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Finite Difference Methods
Mesh Adjustment
Numerical Solution of Differential Equations

20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This second year of research effort has produced a mesh adjustment algorithm for the solution of one-dimensional boundary-value problems based on an arbitrarily given monitor function. A second, more complex algorithm simultaneously finds the solution and the optimal monitor to minimize the truncation error. The results provide an immediately applicable technique for the solution of stiff differential systems such as arise in reactive duct flow problems as well as a firm basis for the extension to the higher dimensional problems of viscous duct flow or external lifting flows.

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I   RESEARCH OBJECTIVES

The general objective of this research is to establish a transformation of coordinates that facilitates the finite difference solution of partial differential equations in two dimensions. The objective can be attained by completion of the following tasks.

A.   Campylotropic Coordinate Definition

A best coordinate net would be composed of orthogonal families of curves on the surface, with the curves of a family coming closer together where the curvature of the curves of the other family increases. The families of lines of curvature have the orthogonality property but may not be the most campylotropic. Analytical effort is warranted to define the best coordinate net.

B.   System Organization

The differential equations form a system that can be presented in various ways. In the one-dimensional case, the given differential equation and the equation for the campylotropic coordinate were combined into a quasilinear system with coefficient determinant properly bounded and bounded away from zero. A rearrangement of the two-dimensional equations into uniquely solvable form needs to be derived.

C.   Numerical Algorithm

Possible methods of approximation of the derivatives by finite differences and solving the resulting algebraic system need to be explored.

D.   Applicability

By analogy with the one-dimensional case, this formidable numerical method is needed only if the solution has regions of sharp variation. The method may be inapplicable to solution surfaces with shock or derivative discontinuities and may also be inferior to a characteristics method for hyperbolic problems. The range of applicability needs investigation.

II STATUS OF THE RESEARCH EFFORT

During this second year the campylotropic mesh adjustment algorithm
for the solution of one-dimensional boundary-value problems has been
generalized and simplified, a mesh adjustment scheme that optimally
satisfies a minimum truncation error criterion has been defined, and
quite direct ways of applying the one-dimensional method to higher
dimensional problems have been developed.

A. Campylotropic Coordinate Definition

The transformation of independent variable to a campylotropic
coordinate, defined by arc length and curvature of the solution curve,
has been shown[1*] to be effective in the finite difference solution of
differential equations of boundary-layer type. The campylotropic coordinate
takes on equally spaced values at the adjusted nodal points. It is therefore
a particular case of the monitor functions defined by White,[2] who showed
that use of properly defined monitor functions provided a convergent
solution method where the Keller box scheme[3] converges.

The conditions to be satisfied by a monitor function so that it
minimizes the sum of the squares of the truncation error at each node
have been reduced to a differential equation and boundary conditions.
Where heuristically suggested monitor functions prove ineffective, it
may be advantageous to add this equation to the system being solved.

B. System Organization

A general way of organizing the given differential equation and
boundary conditions as a first-order system in block tridiagonal form
has been presented by Keller.[3] The transformation to a monitor coordinate
complicates the system by the addition of one differential equation and
by the multiplication of each component of the derivative by a factor.
This added complexity is minor for the high order nonlinear systems,
where the method properly applies.

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*References are listed on page 4.

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C.   Numerical Algorithm

The method is being developed by using finite difference approximations
of low order, to minimize special handling of mesh intervals near the
boundaries. This minimization is increasingly important as the dimension
is raised.

The difference scheme provides a system of nonlinear algebraic
equations. Solution by Newton's iterative method is being implemented,
with full advantage taken of the block tridiagonal form of the Jacobian
matrix.[4]

For problems in two dimensions, the solution region is mapped by
the coordinate change into a region paved with rectangular mesh tiles
and right triangle half tiles. This is the form of domain handled by
subroutine REGION of the ITPACK scheme.[5] Use of this routine or its
general framework will be considered for the geometric part of the
two-dimensional algorithm.

D.   Applicability

The mesh adjustment scheme in one dimension is most suitable for stiff
differential systems with solutions having sharp variations in layers at
a boundary or in the interior. The rate of energy production in steady
flow through a chemical reactor is modeled by such a boundary-value
problem. Sharp variations in reaction rate occur, for example, when short-
lived intermediate species are produced, or where a cold boundary quenches
the reaction.

In higher dimensions, it is expected that the freedom in the co-
ordinate transformation to expand a singular point into a line segment
or to smooth out a boundary layer will be a great aid to computation.

III   PUBLICATION LIST

1.   C. M. Ablow and S. Schechter, "Campylotropic Coordinates," Journal of
      Computational Physics (June 1978).

2.   C. M. Ablow, S. Schechter and W. E. Zwisler, "Node Selection for
      Boundary Value Problems," in preparation.

IV PERSONNEL

Mr. W. E. Zwisler, an exceptionally competent mathematician-programmer, has joined the coinvestigators, Drs. C. M. Ablow and S. Schechter, in the research effort. His biography is attached.

V INTERACTIONS

The contents of the second paper in the publications list will be presented at the fall meeting of S.I.A.M. in Denver, November 12-14, 1979. The abstract is attached.

VI SPECIFIC APPLICATIONS

The one-dimensional algroithm is currently applicable to steady reactive flow calculations used in the analysis of jet energy thrusters, gas generators, or interior ballistics. The two-dimensional algorithm, when completed, should facilitate calculation of steady conical flows for supersonic airframe design, reactive flow fields in shaped axisymmetric ducts, or flows influenced by skin friction and erosion.

VII REFERENCES

1. C. M. Ablow and S. Schechter, "Campylotropic Coordinates," _Journal of Computational Physics_, Vol 27, No. 3 pp. 351-362 (June 1978).

2. A. B. White, Jr., "On the Selection of Equidistributing Meshes for Two-Point Boundary-Value Problems," S.I.A.M. _Journal of Numerical Analysis_, Vol. 16, No. 3, pp. 472-502 (January 1979).

3. H. B. Keller, _Numerical Solution of Two-Point Boundary Value Problems_ (S.I.A.M., Philadelphia, 1976).

4. S. Schechter, "Quasi-Tridiagonal Matrices and Type-Insensitive Difference Equations," _Quarterly of Applied Mathematics_, Vol. 18, No. 3, pp. 285-295 (October 1960).

5. D. R. Kincaid and R. G. Grimes, "Numerical Studies of Several Adoptive Interactive Algorithms," Report CNA126, University of Texas, Austin (August 1977).

WILLIAM H. ZWISLER

Senior Computer Applications Analyst
Computer Resources Group

SPECIALIZED PROFESSIONAL COMPETENCE

Formulation and solution of scientific and data-handling problems,
using large scale digital computing systems; management of the design
and implementation of computer software; management of large scale
computer center

REPRESENTATIVE RESEARCH ASSIGNMENTS AT SRI (since 1964)

Assistant Director, Computer Services

Technical Director, Computer Planning and Operations

Manager, Applications Programming group

Development of computer software, predominantly in FORTRAN and ALGOL for
CDC, IBM, and Burroughs computers

System data analysis and computer programs for Experimental Orbiting Geo-
physical Observatory Satellite; established system to analyze large files
of information and order real-time data, stored time data, and orbital
information

Establishment of system to project employment and population of U.S. cities

Development of TIGER code, a chemical equilibrium and hydrodynamics code

Design of command and control system for U.S. European Command Headquarters

Liaison between Mathematical Sciences Department and other SRI departments
to promote efficient utilization of Institute computing facilities

Implementation of a generalized circuit analysis program

Design and implementation of control optimization program using parallel
tangent gradient method

Testing of virial equation of state for gaseous mixtures

Development of an assembler for the DDP124

Use of data-base management systems

Design and implementation of a large scale logistics information system,
MAGTF

OTHER PROFESSIONAL EXPERIENCE

The Boeing Company: system analysis and implementation for automation of
design of aerospace vehicles

ACADEMIC BACKGROUND

B.S. in engineering physics (1963), University of Washington; M.S. in
statistics (1969), Stanford University; additional training in electrical,
mechanical, and nuclear engineering

NODE SELECTION FOR BOUNDARY VALUE PROBLEMS*

by

C. M. Ablow, S. Schechter, and W. H. Zwisler
SRI International, Menlo Park, California 94025

ABSTRACT

The solutions of two-point boundary-value problems often have narrow
regions of sharp variation, or boundary layers, that can occur in any part
of the interval. A finite difference method of numerical solution will
require more closely spaced nodes in the boundary layers than elsewhere.
An automatic method is needed for achieving the irregular spacing when the
location of the boundary layer is not known in advance. Several automatic
node-insertion or node-movement methods have been proposed. A new node-
movement method is presented that is optimal under the criterion of producing
the least sum of squares of the truncation errors at the nodes. For the
box scheme applied to a system of N coupled first-order differential equa-
tions, this truncation error minimizing (TEM) method increases the system
size to $(N + 6)$ equations. The campylotropic coordinate transformation
method and other published methods based on heuristically derived monitor
functions are node-movement methods that involve systems of only $(N + 1)$
or $(N + 2)$ first-order equations. A comparison is made of the accuracies
of several such methods and the TEM method in the solution of a standard
problem.

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